Algebra 2 – Things to Remember!

CH 8

Exponents:

$x^0 = 1$ $\qquad x^{-m} = \frac{1}{x^m}$

$x^m \cdot x^n = x^{m+n}$ $\qquad (x^n)^m = x^{n \cdot m}$

$\frac{x^m}{x^n} = x^{m-n}$ $\qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$(xy)^n = x^n \cdot y^n$

Complex Numbers:

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ natural log

$e = 2.71828\ldots$

$\log x = \log_{10} x$ common log

Change of base formula:

$\log_b a = \frac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\qquad \log_b 1 = 0$

$\log_b (m \cdot n) = \log_b m + \log_b n$

$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b (m^r) = r \log_b m$

Domain: $\log_b x$ is $x > 0$

Factoring:

Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b + c)$

$x^2 - a^2 = (x - a)(x + a)$

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

Factor by Grouping:

$x^3 + 2x^2 - 3x - 6$

$(x^3 + 2x^2) - (3x + 6)$ group

$x^2(x + 2) - 3(x + 2)$ factor each

$(x^2 - 3)(x + 2)$ factor

Exponentials $e^x = \exp(x)$

$b^x = b^y \rightarrow x = y \quad (b > 0 \text{ and } b \neq 1)$

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x$ $\qquad e^{\ln x} = x$

$\ln e = 1$ $\qquad e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

CH 7

Radicals: Remember to use fractional exponents.

$\sqrt[a]{x} = x^{\frac{1}{a}}$ $\qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{a^n} = a$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: look for perfect powers.

$\sqrt{x^{12} y^{17}} = \sqrt{x^{12} y^{16} y} = x^6 y^8 \sqrt{y}$

$\sqrt[3]{72 x^9 y^8 z^3} = \sqrt[3]{8 \cdot 9 x^9 y^6 y^2 z^3} = 2x^3 y^2 z \sqrt[3]{9y^2}$

Use conjugates to rationalize denominators:

$$\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10 - 5\sqrt{3}}{4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}} = 10 - 5\sqrt{3}$$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$2x - 5\sqrt{x} - 3 = 0 \quad \rightarrow \quad (2x - 3)^2 = (5\sqrt{x})^2$

$4x^2 - 12x + 9 = 25x \quad \rightarrow \; solve: x = 9;\; x = 1/4$

CHECK ANSWERS. Answer only $x = 9$.

CH 9

Working with Rationals (Fractions):

Simplify:

remember to look for a factoring of -1:

$$\frac{3x-1}{1-3x}=\frac{-1(-3x+1)}{1-3x}=-1$$

Add: Get the common denominator.

Factor first if possible:

Multiply and Divide: Factor First

Solving Rational Equations:

Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5}-\frac{3}{2x+1}=\frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ *and get*

$$22-3(x-5)=2(2x+1)$$

$$22-3x+15=4x+2$$

$$37-3x=4x+2$$

$$35=7x$$

$$5=x$$

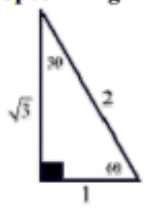
Great! But the only problem is that $x=5$ does not CHECK!!!! There is no solution. Extraneous root.

Motto: Always CHECK ANSWERS.

Trigonometry – Things to Remember!

Arc Length of a Circle = θr (in radians)

Special Right Triangles



30°-60°-90° triangle
side opposite 30° = ½ hypotenuse
side opposite 60° = ½ hypotenuse $\sqrt{3}$

45°-45°-90° triangle
hypotenuse = leg $\sqrt{2}$
leg = ½ hypotenuse $\sqrt{2}$

Law of Sines: uses 2 sides and 2 angles

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$ Has an ambiguous case.

Law of Cosines: uses 3 sides and 1 angle

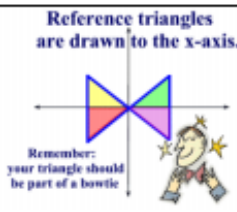
$$c^2=a^2+b^2-2ab\cos C$$

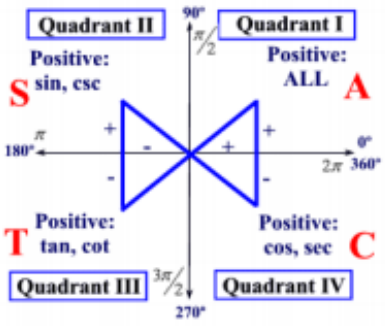
Area of triangle: A = ½ $ab \sin C$

Radians and Degrees

Change to radians multiply by $\frac{\pi}{180}$

Change to degrees multiply by $\frac{180}{\pi}$





Trig Functions

$$\sin\theta=\frac{o}{h};\quad \cos\theta=\frac{a}{h};\quad \tan\theta=\frac{o}{a}$$

$$\csc\theta=\frac{h}{o};\quad \sec\theta=\frac{h}{a};\quad \cot\theta=\frac{a}{o}$$

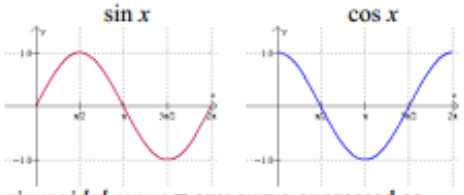
Reciprocal Functions

$$\sin\theta=\frac{1}{\csc\theta};\quad \cos\theta=\frac{1}{\sec\theta};\quad \tan\theta=\frac{1}{\cot\theta}$$

$$\csc\theta=\frac{1}{\sin\theta};\quad \sec\theta=\frac{1}{\cos\theta};\quad \cot\theta=\frac{1}{\tan\theta}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\qquad \cot\theta=\frac{\cos\theta}{\sin\theta}$$

Trig Graphs



sinusoidal curve = any curve expressed as $y = A\sin(B(x-C))+D$

amplitude (A) = ½ | max – min| *(think height)*

period = horizontal length of 1 complete cycle

frequency (B) = number of cycles in 2π (period)

horizontal shift (C) – movement left/right

vertical shift (D) – movement up/down

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{\# of outcomes in } E}{\text{total \# of outcomes in } S}$$

$P(A \text{ and } B) = P(A) \cdot P(B)$
for independent events

$P(A \text{ and } B) = P(A) \cdot P(B|A)$
for dependent events

$P(A') = 1 - P(A)$

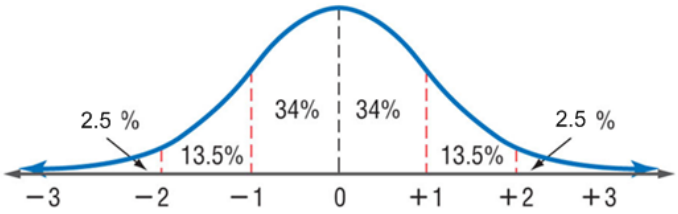
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
for not mutually exclusive

$P(A \text{ or } B) = P(A) + P(B)$
for mutually exclusive

Z-Score - the number of standard deviations a value is from the mean.

$$\text{Z-Score} = \frac{\text{value - mean}}{\text{standard deviation}}$$

KeyConcept Characteristics of the Standard Normal Distribution



- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.
- Centered on the y-axis

So, about 68% of data falls 1 standard deviation from the mean

95% of data falls 2 standard deviations from the mean

z-scores of -2 and 2 are 2 SD from the mean

How to make a box and whiskers plot

1. Order the data.
2. Find the median (**Q2**).

If odd # of values - median is the middle number

If even # of values - median is average of 2 mid values

3. Find the median of the lower half of data - the lower quartile or **Q1**.
   → If even # of values, use all of the numbers in the **lower** half.
4. Find the median of the upper half of the data - the upper quartile or **Q3**.
   → If even # of values, use all of the numbers in the **upper** half.
5. Draw a box and whiskers representation of the data.